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THE ROLE OF TECHNOLOGY IN TIMES OF CRISIS: A CASE STUDY OF CONCEPTUAL GAINS IN DESMOS GEOMETRY LESSONS

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ABSTRACT

In 2020, overnight, educators worldwide saw the need to reinvent the teaching and learning of mathematics since the COVID-19 pandemic spread around the world. This work was carried out at the beginning of the pandemic with five groups of undergraduate elementary education students (here addressed as pre-service teachers or PSTs). The main objective of this research was to investigate how the teaching of mathematics in a completely virtual project could contribute to the development of students' mathematical literacy. The technology used was the software Desmos Geometry and the application Zoom. The methods used in this research were both quantitative and qualitative. Pre and posttests were used to check the students' mathematical gains prior to and after the intervention. The qualitative nature of this research was carried out through the analysis of videos collected via Zoom. The tests used in this study showed that technology contributed significantly to the posttest scores. However, this study also found that the computer did not work as a tutor for students. The technology only fulfilled the function of a tool during this study. The findings of this study suppose that in order to achieve learning, a moderating teacher is required to help students correct their mistakes or clarify doubts during the learning process. The use of videos at the end of each lesson could also help students investigate whether the goals have been achieved.
Keywords: Technology, Mathematical Literacy, Desmos Geometry, Conceptual gains

A FUNÇÃO DA TECNOLOGIA EM TEMPOS DE CRISE: UM ESTUDO-CASO DE GANHOS CONCEITUAIS EM LIÇÕES NO DESMOS GEOMETRIA

RESUMO

Em 2020, da noite para o dia, educadores de todo o mundo viram a necessidade de reinventar o ensino e aprendizagem desde que a pandemia do COVID-19 se espalhou pelo mundo. Este trabalho foi realizado no início da pandemia com cinco grupos de alunos de graduação de pedagogia. O principal objetivo desta pesquisa foi investigar como o ensino de matemática em um projeto completamente virtual poderia colaborar para o desenvolvimento da literacia matemática dos estudantes. A tecnologia usada foi o programa online Desmos Geometria e o aplicativo Zoom. Os métodos utilizados nesta pesquisa tiveram caráter ambos quantitativo e qualitativo. Pré e pós testes foram utilizados para checar os ganhos dos alunos antes e depois da intervenção. O caráter qualitativo desta pesquisa foi realizado por meio de análises dos vídeos

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coletados pelo aplicativo Zoom. Os testes utilizados neste estudo mostraram que a tecnologia contribuiu significativamente para o aumento na pontuação dos pós-testes. Contudo, este estudo também encontrou que o computador não funcionou como tutor para os alunos. A tecnologia somente satisfez a função de ferramenta durante este estudo. Supomos que para alcançar todos os objetivos, seja necessária a presença de um professor moderador que ajude os alunos a corrigirem seus erros ou esclarecer dúvidas durante as lições. O uso de vídeos no final de cada lição também poderia ser usado para ajudar alunos a investigarem se todos os objetivos foram alcançados.
Palavras-chave: Tecnologia, Literacia Matemática, Desmos Geometria, Ganho conceitual

EL PAPEL DE LA TECNOLOGÍA EN TIEMPOS DE CRISIS: UN ESTUDIO DE CASO DE LOS AVANCES CONCEPTUALES EN LAS LECCIONES DE LA GEOMETRÍA A TRAVÉS DE DESMOS

RESUMEN

En 2020, de la noche a la mañana, desde que la pandemia de COVID-19 se extendió por todo el mundo, los educadores de todo el mundo vieron la necesidad de reinventar la enseñanza y el aprendizaje. Este trabajo se realizó al inicio de la pandemia con cinco grupos de estudiantes de pregrado en pedagogía. El objetivo principal de esta investigación fue estudiar cómo la enseñanza de las matemáticas en un proyecto completamente virtual podría contribuir al desarrollo de la competencia matemática de los estudiantes. La tecnología utilizada fue el programa en línea Desmos Geometry y la aplicación Zoom. Los métodos utilizados en esta investigación fueron tanto cuantitativos como cualitativos. Se utilizaron pruebas previas y posteriores para comprobar los logros de los estudiantes antes y después de la intervención. El componente cualitativo de esta investigación se llevó a cabo mediante el análisis de videos recopilados a través de la aplicación Zoom. Las pruebas utilizadas en este estudio mostraron que la tecnología contribuyó significativamente al aumento de los puntajes en la prueba posterior. Sin embargo, este estudio también encontró que la computadora no actuó como tutor para los estudiantes. La tecnología solo satisfizo la función de herramienta durante este estudio. Los resultados suponen que para lograr aprendizaje es necesaria la presencia de un profesor moderador que ayude a los alumnos a corregir sus errores o aclarar dudas durante las lecciones. El uso de videos al final de cada lección también podría usarse para ayudar a los estudiantes a investigar si se han logrado las metas.
Palabras clave: Tecnología, Alfabetización matemática, Geometría Desmos, Avance conceptual

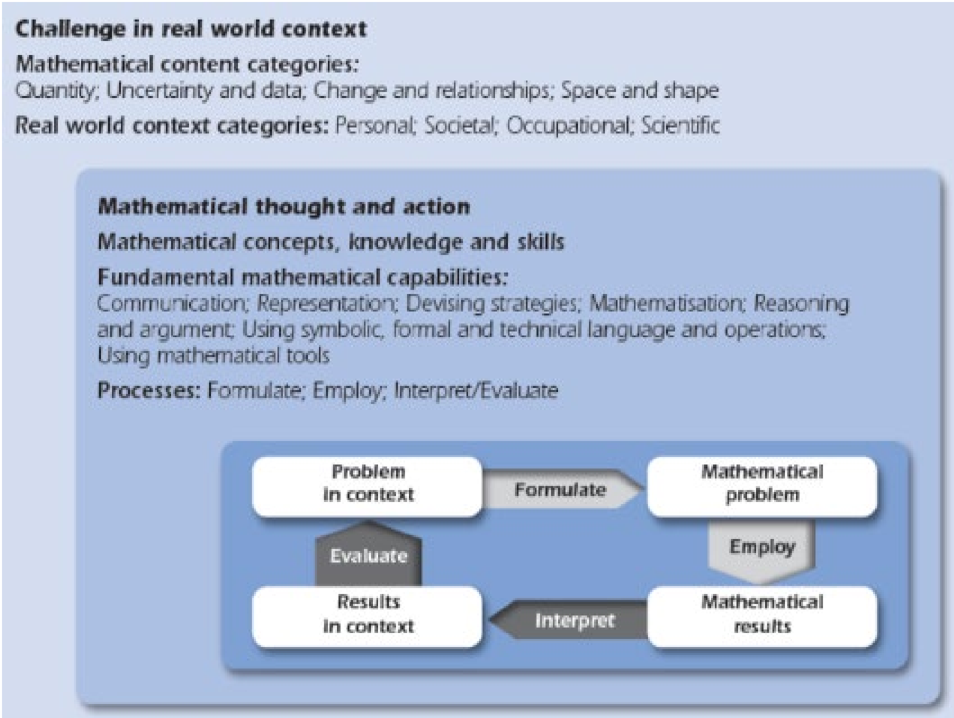
INTRODUCTION

Mathematical literacy is an essential skill for a reflective citizen in an ever-changing world (ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT [OECD], 2016). Mathematical literacy makes it possible to anticipate societal changes (UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION, 2012) and solve real-world problems. For years, educators, policymakers, and researchers have dedicated a large amount of work to make sure students finish their formal schooling as mathematically literate individuals. In such efforts, the OECD conducts the Programme for International Student Assessment (PISA) every three years, which evaluates students on reading, science, and mathematics. Regarding mathematics, PISA is particularly interested in finding out what students know and how they can apply what they know to real-world mathematics problems. PISA's Mathematics results have shown that students from different countries have different performances (SCHLEICHER, 2019). In particular, Asian countries tend to outperform students from non-Asian countries. However, since the PISA Mathematics assessment goal is mathematical literacy, then we can assume there is a disconnect from students' learning to students' outcomes in countries where students do not perform well on the PISA. This disconnect can stem from many variables such as curriculum, teachers, and resources. Focusing on bridging this disconnect can improve students' mathematical literacy. In particular, with the need for the usage of technology in a time of crisis, when schools are not opened across the world and taking into consideration the central role computers play in the "new ways of learning," this study proposes to explore opportunities computers provide to students' mathematical literacy development in contentious times.

Various scholars have researched the usage of technology in mathematics education (see BLUME; HEID, 2008). However, there is a lack of research investigating how technology can improve students' mathematical literacy, with a broader goal of using mathematics to solve real-world problems. Considering PISA's Framework for mathematical literacy (see Figure 1 below), which involves real-world contexts, mathematical concepts, mathematical competencies, and modeling, we hypothesize that mathematical literacy is only achieved if students can conceptually understand mathematical content. Since conceptual understanding is an imperative aspect of the argument being made, this paper draws on theoretical lenses that support analysis from a conceptual standpoint. Regarding the topic for the lessons, it is crucial to consider a topic in the school curriculum across countries and facilitate various interpretations and/or discussions. As one of the most known and discussed topics in mathematics formal schooling, the Pythagorean theorem (see VELJAN, 2000; BRONOWSKI, 2011) was chosen.

The Pythagorean theorem is mostly presented to students through textbooks as the relationship between the legs and the hypotenuses of a right triangle (see Ataide Pinheiro, 2020; TSO, 2011; DANTE, 2010; YEO et al., 2014) without emphasizing conceptually why the theorem works. In other words, students are mostly taught the formula for the theorem. Thus, if students are still taught in traditional ways, there is a possibility that; 1) teachers have a procedural understanding of the theorem; and/or 2) such teaching is a reflection of the curriculum being used. Considering both 1 and 2, this study proposes an investigation on how a five-lesson unit on Desmos Geometry can support pre-service teachers' (PSTs) mathematical gains. Since the Pythagorean theorem is connected to geometrical concepts, this study's theoretical lenses need to encompass both conceptual and geometrical aspects of the Pythagorean theorem. To achieve this goal, this research relies on the framework for figural concepts developed by Fishbein (1993), which is better described later in this paper. This project then looks at the Pythagorean theorem through lenses that review both a conceptual and a figural perspective.

Figure 1 – PISA model for mathematical literacy. Adapted from PISA 2015 assessment and analytical framework: Science, reading, mathematics, and financial literacy



Source: OECD (2016). Paris, France: Author. Copyright 2016 by OECD

The technology approach in this study takes upon the understanding of a dual perspective on the theorem in question and expands to reflect ways that the computer as a tool and tutor (PEA, 1985) can influence the development of PSTs' conceptual understanding. This research hypothesizes that learning mathematical topics conceptually leads to better teaching of such topics. This study also tests how a completely online teaching would look like in times where schools can't be opened. Since this research's ultimate goal is helping PSTs develop their mathematical literacy, a conceptual framework for the understanding of mathematical activities is used as well (see HEID; BLUME, 2008).

LITERATURE REVIEW

This project was piloted by an investigation from the first author on why students from different countries learned mathematics in different ways, therefore, performing differently. In particular, mathematics textbook questions were assessed according to their level of cognitive demand, mathematical competencies, and mathematical modeling required in those tasks (Ataide Pinheiro, 2020). Also, a PISA-based exam was developed to assess Brazil and Taiwan students in solving questions like the ones in the PISA exam. In both countries, students had access to a four-operation calculator. The findings in that study were, 1) Taiwanese students, just as in PISA, outperformed Brazilian students, and surprisingly 2) in Taiwan, students did not need the calculator to solve the problems in the exam, yet they still performed much higher than Brazilian students. Technology, in such cases, was not an influence on students' performance. However, going back to mathematical literacy, one could question: to what extent the usage of different technologies with different purposes could support students to improve their mathematical concepts so that they can learn mathematics conceptually and apply their mathematical knowledge to solve real-world problems?

Pythagorean Theorem

The Pythagorean theorem is a familiar topic for most people who ever studied geometry or algebra in their lives across the world. As Veljan (2000) argues, the theorem can be the only nontrivial mathematical theorem that some people know by heart. Great mathematicians came up with excellent results in mathematics using applications of the Pythagorean theorem. In the Fermat's Last Theorem, the Diophantine equation $(a^n + b^n = c^n, n \geq 3, abc \neq 0)$ was said by Fermat to have no solution in 1650. However, in 1995, the mathematician A. Wiles proved that actually, this equation has solution for $n \geq 3$. This theorem has also been used by the second most prolific mathematicians of all time (Euler remains the most prolific mathematician of all times) the Hungarian Pál Erdõs. Erdõs published over 1500 papers in applied mathematics areas such as discrete, combinatorial, and computational geometry. In most of his research, he relied on the Pythagorean theorem to solve mathematical problems. These applications not only stayed in the realm of intra-mathematical problems, but it also extended to real-world situations that people encountered in their everyday lives.

The Pythagorean theorem is one of the most proved theorems of the history of mathematics, if not the most proved, having more than four hundred different known proofs (MAOR, 2007), and is also considered the most famous theorem of mathematics. This flexibility on ways to be proved is also an important aspect of the theorem, since its proof, in many cases, are closely related to the understanding of the basic concept behind the theorem, area of squares. In his book *The Ascent of Man*, Bronowski stated that the Pythagorean theorem is undoubtedly the most important in the field of mathematics (BRONOWSKI, 2011). Finally, Pythagorean Theorem is also part of one of the most translated and copied books (except the Holy Bible) in the history of humanity, the Elements of Euclid (HEATH, 1956). We ackowledge that although the theorem was named after Pythagoras (582-507 B.C.), this relationship was understood much earlier by Chinese and Egyptian geometers (Aichele & Wolfe, 2008, p. 317).

Role of Technology

Research has shown that technology is an important tool for teaching and learning mathematics in this rapidly developing world in which we live. Clements, Sarama, Yelland, and Glass (2008) found that computer environments such as Logo, Cabri Geometry, and Geometer's Sketchpad can support the teaching and learning of geometry for elementary and middle school students in new and dynamic ways, which can also influence the "traditional" ways of teaching geometry. In addition, Hollebrands, Laborde, and SträBer (2008) conducted a literature review on studies that used technology to support the teaching and learning of geometry at the secondary level. They found that 1) a variety of studies analyzed how technology functions as a window in students understandings and their construction of meanings; technology can be used to allow students to approach the content from a perspective that is different than their own, breaking the traditional way to do geometry, and 2) many studies focus on technology as a tool capable of shaping students' strategies towards mathematics. They also called special attention to the importance of studies that emphasize the role of technology in proving mathematics. The studies above demonstrate the importance and the many contributions technology plays in students' learning.

In this study, the computer is used as a tool that administers tasks one at a time to the students and facilitates the learning of the theorem by sequentially organized lessons. This research borrows the concepts defined by Taylor (1980), in which computers are seen as *Tutor*, *Tool*, and *Tutee*. In this project, in particular, computers are envisioned as both tool and tutor.

Possibilities Afforded by Technology

Heid and Blume's (2008) research found that technology can afford changes in students' ways of conceptualizing, symbolizing, representing, and generalizing mathematical knowledge. In their work, they found that technology can influence the ways students learn algebra contents. They also found that technology works as a catalyzer for the learning of algebra through the introduction of new content and targeting different understandings of traditional algebra. Heid and Blume's work with Algebra envisions the usefulness of their conceptual framework. Desmos Geometry, a cognitive technology (PEA, 1985), will be used as an organizational and visual tool influenced by the instrumental genesis. As defined by Pea (1985), this organizational aspect, and envisioned in this study, positions the technology as both an amplifier and a reorganizational tool. Desmos will give students opportunities to amplify their ability to determine the area of squares in non-standard ways through a series of problems in various contexts. Dragging and drawing (reorganizing students' mental activities) and erasing is extended on the screen for fast (capitalizer) and empirical mathematical activities. Such activities are transmitted through cognitive technology as content and tasks. It thus facilitates student's representation of areas, the conceptualization of conjectures and relationships with symbols, finally getting to the generalization of the relationship in question, the Pythagorean theorem. All these mathematical activities combined will generate the development of concepts and procedural skills, consequently achieving the 5-lesson unit project's goal.

In conclusion, we envision the connection between theory and research in this project. The theoretical perspectives that lead us to the usage of Desmos Geometry to develop the five-lesson units come from the inspirational work developed by Fishbein (1993). Fishbein defined that each geometrical concept is intrinsically determined by two components, 1) a figural, and 2) a conceptual. A right triangle for many students only holds the figural components, and the relationships behind the figure (image) cannot be achieved. With Desmos Geometry, we envision helping students be equipped with both the figural and the conceptual components of geometrical figures, therefore, providing gains in these students' knowledge of the Pythagorean theorem.

METHODS AND METHODOLOGIES

This study aimed to implement five collaborative Desmos Geometry lessons on the Pythagorean theorem to groups of three to five students via Zoom. These sequenced lessons had the intention to support students' development of a conceptual understanding of the Pythagorean theorem. At the primary stage of this study, the intervention was applied to undergraduate elementary pre-service teachers (PSTs) from a large research institution in the Midwest of the United States. PSTs were fundamental for this study since we hypothesized that their understanding of the Pythagorean theorem would not go beyond procedural fluency, reflecting current formal schooling and curricula (see DANTE, 2010; TSO, 2011, YEO et al., 2014). In the future, the instructional unit used in this study will be revised and administered for 8-9th grade students. These grades were chosen specifically because they correspond to the time when the Common Core State Standards (CCSS) Initiative (2010) requires students to be able to apply the Pythagorean theorem in two and three-dimensions problems to determine the unknown side lengths of right triangles.

The PSTs completed a mandatory pretest that pinpointed PSTs' understanding of the Pythagorean theorem before receiving any kind of intervention (five-lesson unit). PSTs were also asked to not use any kind of external materials during the pretest, which includes no usage of internet, textbook, calculators, etc. While it was hard to fully control these rules, since the whole interaction was online and students were not held accountable for their scores, data were analyzed supposing that they were strictly followed.

Finally, after the PSTs' had gone through the five lessons, they were invited to take a posttest. Both the pretest and posttest were very similar. They contained the same number of problems, similar questions, but with different quantities for each question. PSTs were once again asked not to use any kind of external resources while they answered the posttest. This final phase of the study provided data on how students gained knowledge of the Pythagorean theorem by being part of the study.

The five-lesson instructional units were conducted in small groups. There were multiple reasons for this approach, in particular, the role collaborative learning has on students' conceptual gains (KYNDT et al., 2013; LOU et al., 1996; SLAVIN, 1980).

As stated by the OECD, the current workforce demands people to work together and solve problems with others; thus, collaborative problem-solving should be encouraged (OECD, 2017). The CCSS is also built on the principals of having mathematical problem-solving and collaboration interwoven across the standards (COMMON CORE STATE STANDARDS INITIATIVE, 2010). Finally, this study's authors envision mathematical classrooms as places of a community of learners in which mathematical knowledge is built through collaboration. We believe in student-centered instruction where students work together and have different roles within their groups, and knowledge is constructed through their discussions, questionings, and argumentation of the problems (see FORMAN, 1992; BENBUNAN-FICH; ARBAUGH, 2006; TOOMELA, 2007). With such, students were also assigned different group roles to work collaboratively and effectively in their mathematical discussion.

Brief Descriptions of the Research Instruments

The instruments used in this study were developed collaboratively among both authors. All this collaboration was done via Zoom meetings, phone calls, emails, and text messages.

Tests

The tests, which were very similar to each other, were developed to assess an overview of PSTs' understanding of the Pythagorean theorem. Since we were interested in knowing whether there were gains to PSTs' knowledge on the Pythagorean theorem after an intervention (five-lessons unit), it was essential to develop a test that aligned with the content addressed in the lessons. In no moment during our lessons, the

Pythagorean theorem and its formula were mentioned. Therefore, the lessons were an approach to the learning of the theorem through an inquiry-based teaching style.

The tests were composed of 8 questions each and are discussed with details in this section. The first question asked PSTs if they knew of any relationship between the sides of a right triangle; here, we were interested in knowing if they were going to mention the theorem at all, and in understanding how they connected legs, hypotenuse, and right triangles to the Pythagorean theorem. The second item asked PSTs if they were able to prove the Pythagorean theorem by a given picture that is famously used in textbooks worldwide for the proof of the theorem. We had that item as the very second one because, for us, it is only reasonable for students to use the theorem if they can guarantee such a theorem indeed works. The third item required students to connect the idea of the area of squares formed on the legs and hypotenuse and see the relationship between them. As such, PSTs would know that the area of the square that had side length the same as the length of the hypotenuse equaled the sum of the areas of the squares formed with sides equal the length of each leg. Note that once again, this question did not provide a formula; rather, we were interested in the students' conceptual understanding of the theorem. The conjecture built through problem three was the one worked with PSTs throughout the unit. Problem four and five directly presented to students a right triangle with a missing side length. For these problems, we envisioned students using the relationship they found, which could be using the formula or thinking about the area that could be constructed on top of those legs and hypotenuse. Problem 6 was the inverse of the Pythagorean theorem, in which when given three sides, would those sides make a right triangle? We expected PSTs to consider the conditions found on their conjecture that relates the three sides of a right triangle. Question seven was an application of the Pythagorean theorem to finding the distance between two points in a coordinate plane. Furthermore, our last item was the application of the Pythagorean theorem into a real-world problem.

Lessons

Using Desmos Geometry as a tool and tutor provided ample opportunities for 1) sequential lessons for PSTs on the Pythagorean theorem; 2) a place for visual understanding of geometry; and 3) an interactive tool that could be used remotely in times of crises such as the ones we found ourselves during the COVID-19 pandemic. Our unit was composed of five lessons presented in detail in Appendix A.

Data Analysis

This study used both quantitative and qualitative methodological approaches for the interpretation of the data collected.

Test Analyses

To make sense of the data collected via the tests, we used descriptive statistics. It was also needed to use a specific framework that helped us interpret and understand the test scores' meaning. Scores were created according to the level of cognitive demand that each item represented. Stein and Smith (1998) defined four approaches to illustrate the cognitive demands of mathematical procedures. Memorizing facts without connecting them to other knowledge imposes fewer cognitive demands. Conceptual thought processes involved in formulating connections to mathematical meanings or exploring relationships among concepts impose more cognitive demands, and open the door to deeper learning. For this study, tasks of lower cognitive demand are those that involve direct utilization of the conjecture or theorem; i.e., given two sides of a right triangle, determine the third one. Tasks of higher cognitive demand are those that involve the interpretation of real-world situations, connection to multiple mathematical knowledge, as well as multi-step problems, i.e., to find the length of a side, you need to interpret first the area of the given square (see Ataide Pinheir, 2020; TSO et al., 2018). Having in mind that not all questions requested from PSTs have the same level of cognitive demand, we graded questions according to their level, which here was divided into low-level and high-level. Also, each item was graded as full credit, in case the PST answered the item correctly, partial credit, if the answer was correct but not fully answered or articulated, or answers that were incorrect but made sense because of some misinterpretation of the conjectures and theorem, and finally, items that were answered incorrectly were graded as no credit and received a score of 0. Because of challenges brought by the pandemic, only 20 PSTs responded to the pretest and 18 to both pre and posttests (N=18). A full rubric for the coding scheme for the cognitive demand of the questions as well as a rubric for the grading of each problem is presented for the pretest in Appendix B. The same coding scheme and rubric were used for the posttest, since both instruments assessed the same but include different quantities for their problems. High-level cognitive demand items had a weight twice as much when compared with low-level cognitive demand ones. Therefore, if a high-level cognitive demand item was worth four points, a low-level cognitive demand item was worth two points when correctly answered.

Lesson Analyses

The Desmos Geometry five-lesson unit were analyzed using a qualitative methodological approach. Because of space limitation, we focus this paper only on two of the five groups of students. The findings related to these two groups reflected what happened across the five groups in general. Using the software MAXQDA, the collaborative lessons were coded separately by both authors. Then, a discussion occurred to frame the coding book and decide which codes were relevant for this study.

FINDINGS

Tests

PSTs' scores increased 34.63% from pre to posttest. The pre and posttests were used only to verify if PST's were having gains in mathematical conceptual knowledge while taking the lessons. While the number of participants (N=18) who took and completed the tests is still low, we anticipate that PSTs' scores have shown a significant increase. In table 1 below, we show an overall comparison of PSTs' grades from their pre and posttest.

Table 1 – Overall PSTs' scores

Pretest	Posttest	increment
14.61	19.67	34.63%

Source: created by the authors

This paper analyzes the data for the unit of two specific groups, here called group 1 and 2. Table 2 below presents the average of the scores obtained by PSTs on the pre and posttest as well as the percentage increment. These two groups' data are analyzed since they reflect what happened across all the five groups that participated in the study and give this study an understanding of things that came up during the unit lessons and the learning opportunities obtained by participants in these groups.

Table 2 – Average grades of group 1 and 2 and increment rate

	Pretest	Posttest	increment
Group 1	6.67	12.25	83.66%
Group 2	12.3	18	46.34%

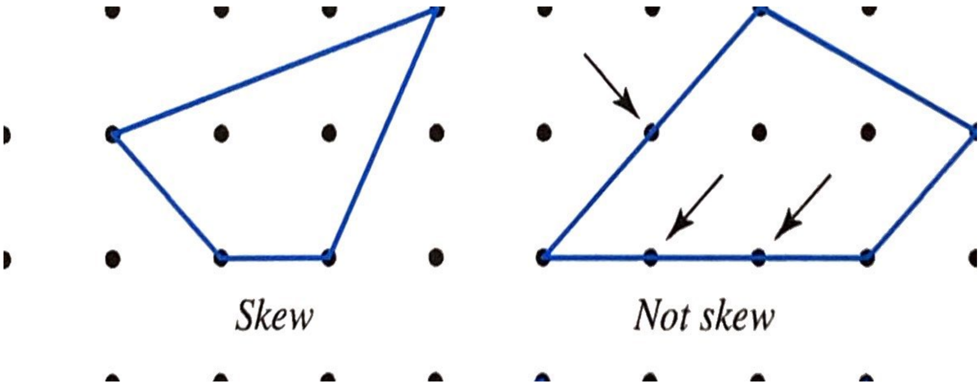
Fonte: created by the authors

Unit

Group 1

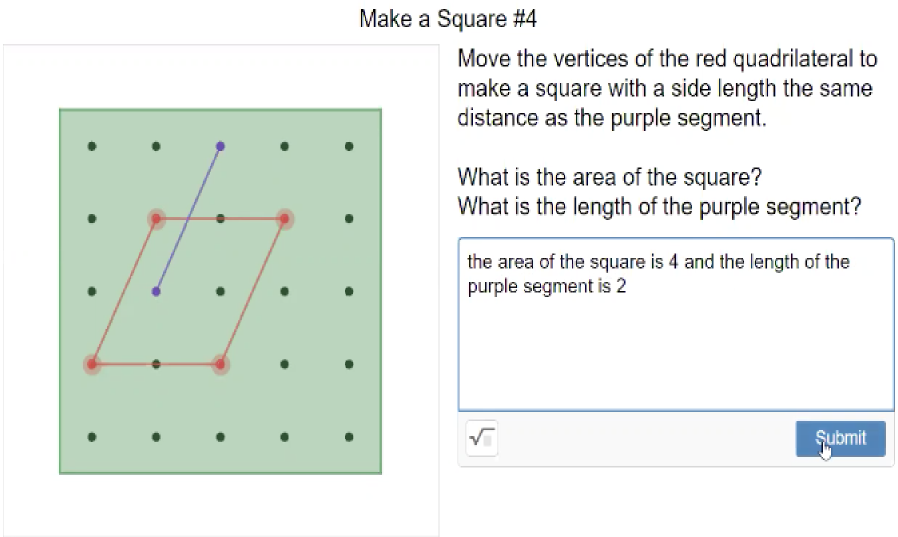
Group 1 went through the five lessons quickly, in the timestamp of 1h17min. The PSTs described the areas of the figures by using area formulas; for example, the square area is side times side, while the area of a triangle is base times height divided by two. By the end of the first lesson, PSTs were able to determine the area and the length of non-skewed figures – geoboard figures that edges touch more than two pegs at a time (Aichele & Wolfe, 2008, p.241); however, they failed to determine the area and the skewed figure's side length, as shown in figure 3 below – skewed figure as those in which each segment touches exactly two points at each end (Aichele & Wolfe, 2008, p.241.

Figure 2 – Example of skewed and non-skewed figures



Source: Aichele & Wolfe (2008)

Figure 3 – Students' answer for the area and length of a skewed square

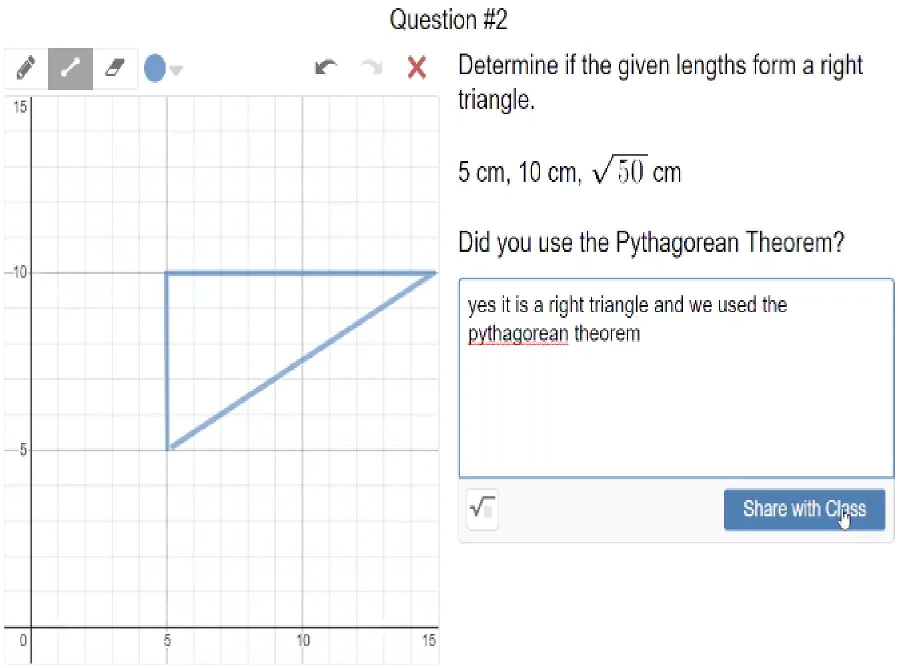


Source: PSTs' work

Because of this significant misunderstanding of areas of skewed squares, PSTs were unable to notice a pattern in the table they created and determine a conjecture for right triangles. Because no conjecture was made, the proof of the theorem also did not make sense for them. At no moment were they able to justify why the theorem works the way it does. They also mentioned that they wish they had an instructor teaching them about these "problems" first, rather than just being given specific tasks to be completed. We assume that the computer was not tutoring this group of PSTs. Instead, they were using the computer only as a tool throughout the activities.

When PSTs started lesson 4, where they were asked to use the Pythagorean theorem, they relied mostly on their understanding of the formula. While they found out missing lengths on right triangles, they slowly relied on the Pythagorean theorem formula. They stated the formula and used it to solve all the problems. One exciting part of this lesson occurred when they were asked if the side lengths 5, 10, and $\sqrt{50}$ formed a right triangle. Automatically, students drew on the e-dot paper a right triangle with legs of length 5 and 10 and assumed $\sqrt{50}$ was the hypotenuse and said they used the Pythagorean theorem to solve it. If they had used the theorem, they would notice that 25+100 is not equal to $\sqrt{50}$. They assumed that the $\sqrt{50}$ was the hypotenuse and did not stop to think that 10 was actually the largest number.

Figure 4 – Mistakenly using the theorem and getting to a wrong answer



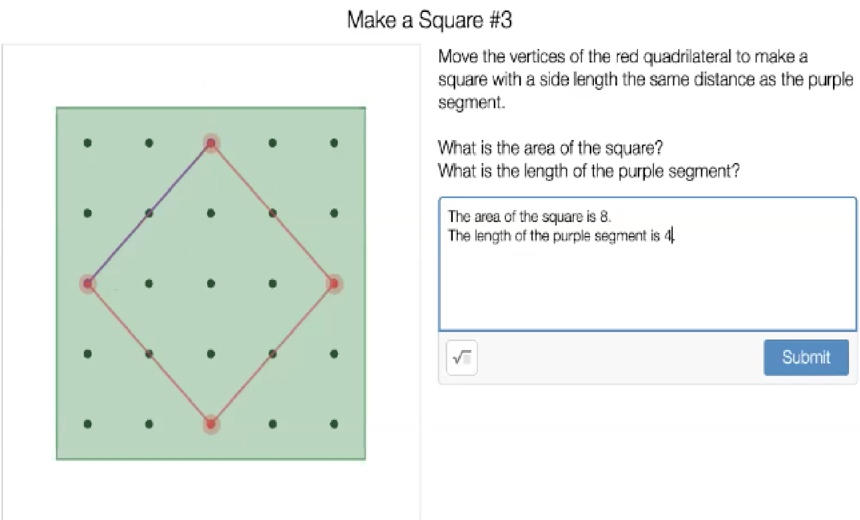
Source: PSTs' work

Another exciting episode that shows evidence of PST's relying heavily on procedures learned in previous geometry courses comes from a problem in finding the distance between two points on a coordinate plane. When posed with the question, one of the PSTs automatically said, "isn't it rise over run?" such a statement is commonly used in textbooks in the U.S. to calculate slopes. Finally, on real-world mathematics problems, PSTs spent much more time trying to figure out the answers. However, after lengthy discussions, they were able to come up with the right answers. Mostly, they relied on the Pythagorean theorem formula to find the missing side of right triangles.

Group 2

Like group 1, group 2 also used formulas that they remembered to determine the area of the figures in Lesson 1. The only difference between group 1 was that group 2 was able to determine the area for both skewed and non-skewed figures. However, group 2 was not able to determine the purple side provided on the e-dot paper. They simply divided the square area by two to state what the length of the square was. In other words, if the area was 8, the length was 8/2=4, as shown in figure 5 below. When working on the next problem, the PSTs were going to commit the same mistake but noticed that what they were doing was not correct because the area was not totaling to the number it should. For example, in problem #3, 4x4 is not equal to 8; therefore, 4 cannot be the square's side.

Figure 5 – Mistakenly interpreting the purple side



Source: PSTs' work

After going through lesson 2, the PSTs concluded that the Pythagorean theorem, here also referred to by PSTs as "a formula", worked for any kind of triangle. Even though the quantities they had in their tables would not demonstrate the same, PSTs were confident that the theorem worked for acute, obtuse, and right triangles. One of the PSTs questioned at some point in lesson 2 on whether the theorem wasn't just applied to right triangles; however, one of the leading PSTs for that group convinced the group that the theorem would work for any kind of triangle. Figure 6 below shows how their conjecture could not be confirmed by the quantities they provided.

Figure 6 – Group 2 attempt to provide a conjecture

Making a Conjecture

Triangle	Area of Blue Square	Area of Green Square	Area of Purple Square
Acute	36	18	18
Acute	5	8	9
Right	9	16	25
Right	4	4	6
Obtuse	13	55	68
Obtuse	169	45	45

Record the areas of the squares on the 6 triangles that you have made observations about in the table. What relationship exists among the areas of the squares? Is it the same for all types of triangles?

If you square all of the sides and then add the two smaller sides (numbers) together, you will get the bigger number. This is the Pythagorean theorem. This is the same for all types of triangles.

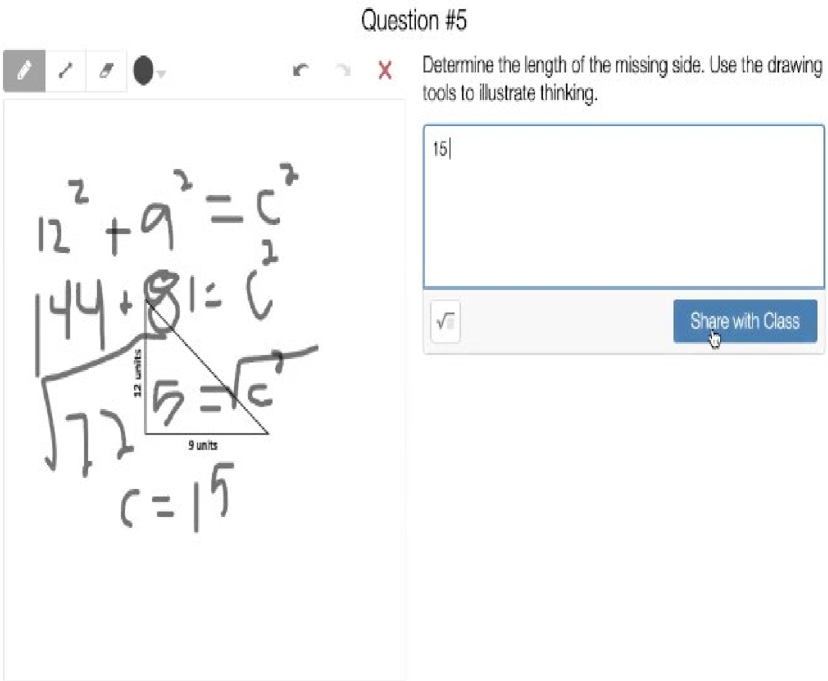
Edit my response

Source: PSTs' work

At the beginning of lesson 3, one of the PSTs cheated and checked the internet and told the group the Pythagorean theorem only worked for right triangles. When trying to prove the Pythagorean theorem on lesson 3, group 2 could not use the area and the relationships between areas and sides of a triangle to prove. They got much closer than group 1. They were able to see that the two big squares had the same areas (see question #2 on the pretest in Appendix B) and that the purple square area was the same as the area of the blue plus the area of the green square. They got to that because they saw that each big square had four similar orange triangles. Therefore, the purple area needed to be equal to the sum of the blue plus the green area.

Even when asked to draw, both groups 1 and 2 wrote a formula of the theorem instead, as shown in figure 7 below. At no moment, PSTs were able to connect the theorem with the area of squares. On the contrary, PSTs relied solely on the formula to solve the proposed problems in lessons 4 and 5.

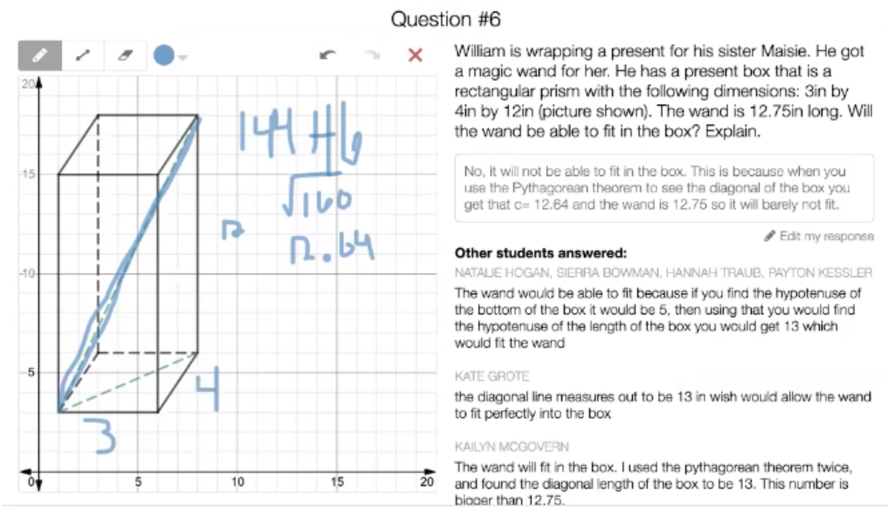
Figure 7 – Drawing formula instead of drawing squares



Source: PSTs' work

While group 2 did not spend too much time on question #5 (a real-world problem) because one of the PSTs had already figured out when she solved the pretest, they mistakenly answered question #6. As shown on the picture below, they found the hypotenuse for the right triangle with sides 12 and 4. They could not correctly interpret the problem. Even after checking other PSTs' answers, they were convinced they were right and decided to finish the lesson.

Figure 8 – Group 2 solution for question #6



Source: PSTs' work

DISCUSSION AND CONCLUSION

While Desmos Geometry showed significant gains for students in their posttest scores, it became evident that students from both groups 1 and 2 did not learn the theorem as we proposed. In no moment during the five-lessons were PSTs able to build a conjecture for the Pythagorean theorem by connecting the relationship between lengths and areas of a right triangle, nor were the PSTs able to present a complete proof of the theorem. Mostly, PSTs discussed the formula they knew for the theorem and tried to use the formula to solve the problems proposed in lessons 4 and 5. Group 2 was more successful than group 1 in the lessons and also on the tests' scores. Desmos Geometry provided the opportunity for PSTs to inquire together as they explored the Pythagorean theorem. PSTs in these groups also had a hard time finding areas of skewed figures. Such an issue was never completely solved for group 1, even though group 2 figured it out after some attempts.

Reflecting on what happened to groups 1 and 2, we assume that these challenges could be solved if we had the lessons implemented with a teacher present during PSTs' interaction because we would discuss the answers they found throughout the lessons. For example, when students found the area of a figure by using the formula in the first lesson, an intervention through discussion could help students discover other ways to find the area. Another example is, when students got stuck discussing the area of skewed figures, a classroom discussion could have deepen this understanding before moving on to the next lesson where they created a table of areas. In most cases, the data was not correct. Such interventions could be done by a moderator teacher and would avoid students progressing in the lesson without having an understanding of the intended content from that lesson.

Desmos geometry could not function as a tutor to support students' development of a conceptual understanding of the Pythagorean theorem, as we envisioned at the beginning of this paper. Mostly because we did not anticipate that PSTs' would have a very low understanding of area, resulting in the non-development of a conceptual understanding of the theorem. It became clear by the end of the lessons that the PSTs were not able to establish a clear connection between conceptual and figural components of geometric concepts (mainly area), as imagined and theorized on Fishbein' (1993) figural concepts. However, Desmos served as a tool for PSTs to explore mathematical concepts. Desmos failed to support group discussions among students who were confident with the formula but lacked conceptual understanding of the formulas. One way this could have been improved is by providing student's answers and not letting them move forward without having correct answers. However, this emphasises a traditional style of teaching. Our ultimate goal for mathematical literacy was not achieved for groups 1 and 2, as hypothesized at the beginning of the paper. A student is only mathematically literate if they can conceptually understand the meanings behind the formula they are using. Neither group understood and explained why the theorem worked the way it does, making us conclude that PSTs merely assumed the Pythagorean theorem was true.

Connecting back to the framework for mathematical activities (HEID; BLUME, 2008), PSTs used Desmos Geometry on mathematical activities. The technology helped PSTs to develop their representation and symbolic work. This study did not provide much evidence on the gains the technology afforded for the development of concepts. However, the technology did provide PSTs with opportunities for the development of skills and procedures. For such, we attributed the gains PSTs have on the posttest. While these gains were relevant, as discussed above, they did not support PSTs development of their mathematical literacy.

Not all PSTs from group 1 and group 2 were equally participating in the lessons. In group 1, some PSTs seemed not to be very clear about how the answers were being generated, and others did not contribute to the discussion throughout the five lessons. In some groups, a couple of PSTs lead the discussion the most. In group 2, a PST played a significant role as she questioned what the other PSTs were doing to find the answers. Therefore, assigning strict roles for each participant was essential to keep students engaged with the content they were learning; however, it was still hard to make sure PSTs were fully engaged in their roles since no teacher moderator was overlooking the lessons.

Overall, this research study found that Desmos Geometry may be used to improve PSTs conceptual understanding of the Pythagorean theorem, but not the way it was implemented, by solely using the well-sequenced lessons. We envision that a teacher moderator or videos embedded in the lessons are needed during the lessons to better guide PSTs on the specific goals of each activity. This is an essential remark as we move forward during the COVID-19 times and cannot be inside the classroom with students. While Desmos plus Zoom provided an excellent tool for collaboration, since PSTs could work on their own and still see each others' screen, the lessons failed to guide PSTs to develop their concepts, inhibiting the ultimate goal of this study, mathematical literacy.

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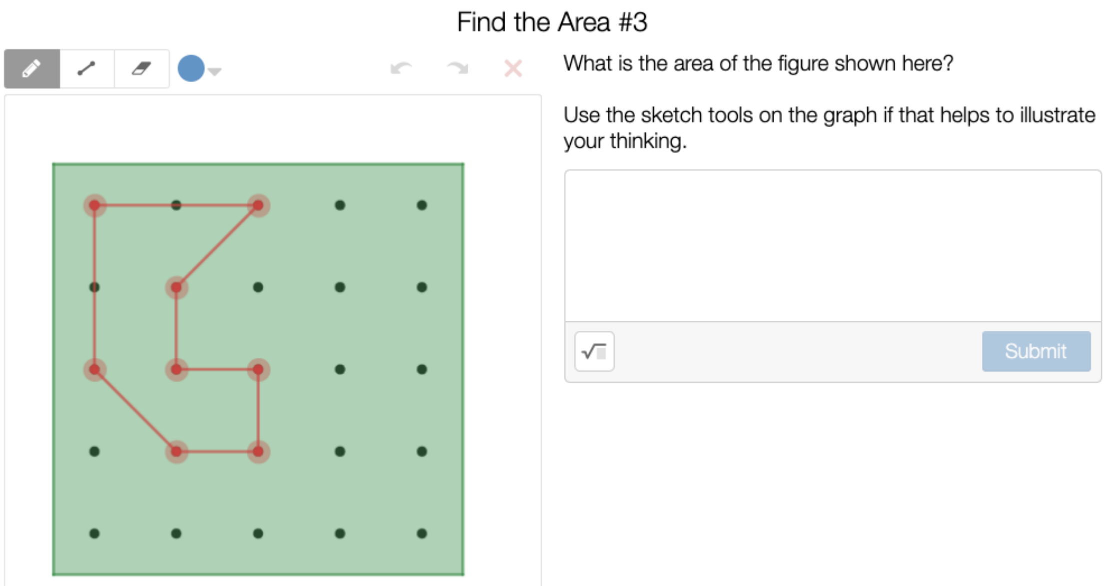
APPENDIX A

Lesson 1

Our first lesson was entitled "Area and Squares." In this lesson, students are provided with the opportunity to develop their understanding of the area on an e-dot paper (electrotonic dot paper). While in no moment we defined what area means, we structured it so that students can count square units on a geoboard to express the area of a series of images. As defined in Geometry Structures by Aichele and Wolfe (2008), in a geoboard, the vertical and horizontal distance between two dots is considered to be 1 unit of length; therefore, one square unit in the area of a square with sides of 1 length unit. One example of an activity PSTs was proposed to work on this lesson is provided by figure 2 below.

Figure: Example of a problem proposed in Lesson 1



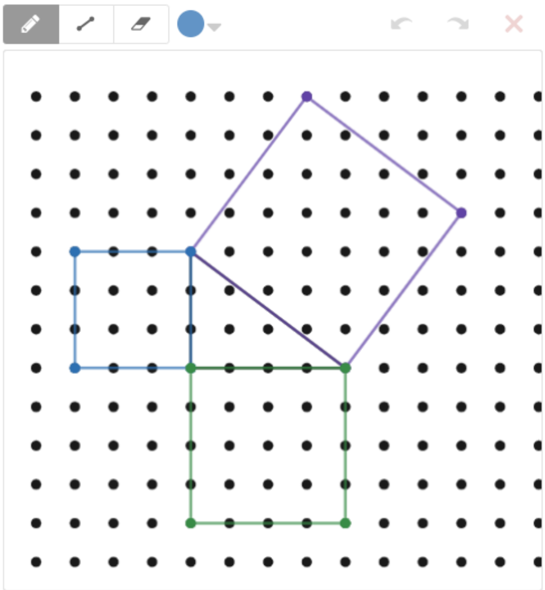
Towards the end of Lesson 1, we also provided PSTs with activities that investigated the relationship between the side of a square and its respective area.

Lesson 2

The second lesson was entitled "Discovering the Pythagorean Theorem." In this lesson, PSTs were allowed to explore relationships between the areas formed with the sides of different triangles' side length. We provided acute, right, and obtuse triangles definitions for students, and squares with sides congruent to those of each different type of triangles. After working with various activities comparing the areas, PSTs were asked what relationship existed among the areas of the different squares formed along each side of the provided triangles. The goal of this lesson was for PSTs to find that in a right triangle, the area of the square with side length congruent to the length of the hypotenuse is the same as the sum of the area of the two other squares that have side lengths equals to the length of those legs. One example of an activity from Lesson 2 is provided in figure 3 below. At the end of lesson 2, we asked students to construct the right triangles on their e-dot paper and test their conjecture. It was crucial for us to have students explore their conjecture in this lesson because such conjectures would be fundamental for proving the theorem. In other words, students explored the relationship between the area of squares and legs and hypotenuse of a right triangle in particular cases and were able to build a conjecture that the area of the square with side length equal to the hypotenuse is the same as the sum of squares with side length the same as the legs of the right triangle, but is such statement valid for any case of just the cases investigated? This is an answer for lesson 3 discussed in the next section.

Figure 3: Example of activity from Lesson 3

Explore Right Triangle



A right triangle is a triangle with one angle that is 90 degrees.

In the figure to the left, there is a right triangle. On each side, there is a square that has the side length the same as the length of each side of the triangle. Find the area of each square.

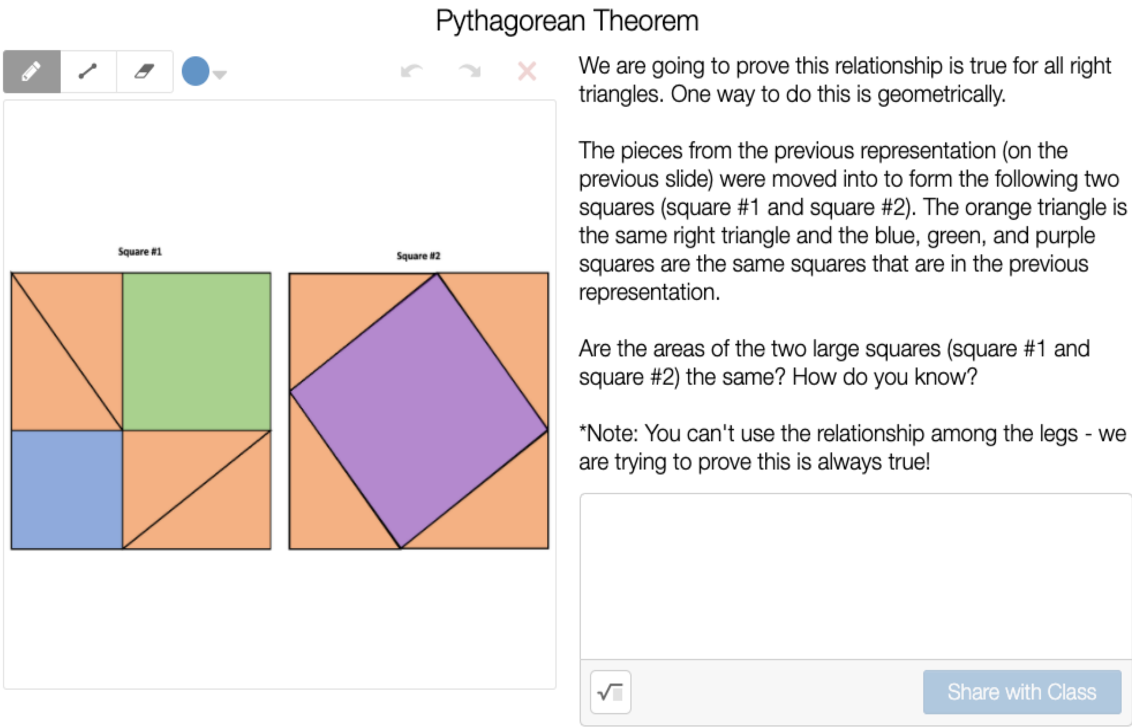
*Recorder #2: Be sure to record these numbers for later too!

Triangle	Area of Blue Square	Area of Green Square	Area of Purple Square
Right			

Lesson 3

This lesson was fundamental in this instructional unit project. Desmos Geometry gave us an excellent opportunity to organize sequentially well the activities we wanted PSTs to work with. After this lesson, it would make sense for PSTs to use the theorem to solve problems involving right triangles because they would have proved that such a theorem was going to be valid for any right triangle. In this lesson, PSTs explored one way to prove the Pythagorean theorem. We provided them with a picture of the right triangles with sides a, b, and c. Then, we asked the relationship between the triangle's legs and the hypotenuse based on the conjecture they build on Lesson 2. Followed by this activity, we proposed a guided geometric proof. Figure 4 below provided for PSTs a guide for the proof of the theorem. PSTs were supposed to explore the length and the area, then find whether or not the two big squares were the same. After finding that the two big squares' area was the same, they were provided with the opportunity to explore the relationship of the purple, blue, and green squares. Their conclusion should lead them to the understanding that the purple square area, with side c, was equal to the sum of the areas of the square with side length b, and the other square with side length c.

Figure 4: Picture provide to students as support to prove the Pythagorean theorem

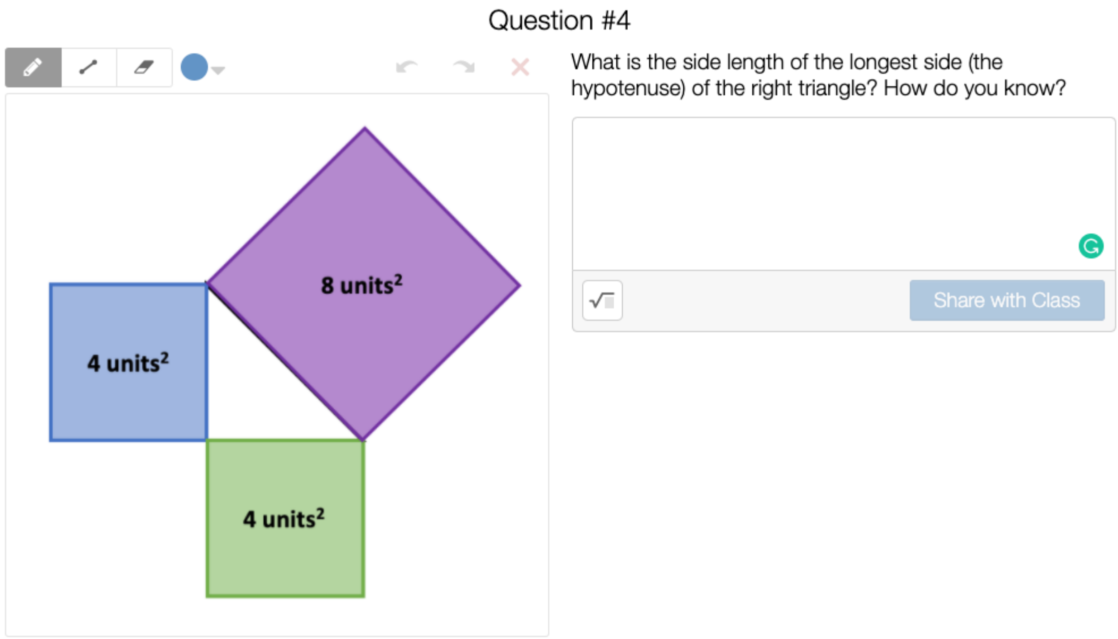


Such can be generalized as the general formula for the theorem, a2+b2=c2.

Lesson 4

This lesson departures from the assumption that PSTs were able to prove the Pythagorean theorem and now they can use the theorem to solve any problem that is related to right triangles, since they proved the theorem works for any right triangle. Lesson 4 provides PSTs with three kind of questions: 1) provide the area of squares on top of legs and hypotenuses and ask students to determine the area of the missing square, 2) provide students with the area of squares and ask them to determine the length of a side, as demonstrated in figure 5 below, and 3) provide the length of two sides of right triangle and ask PSTs to find the third one. We hope PSTs connect back to their conjecture from Lesson 2.

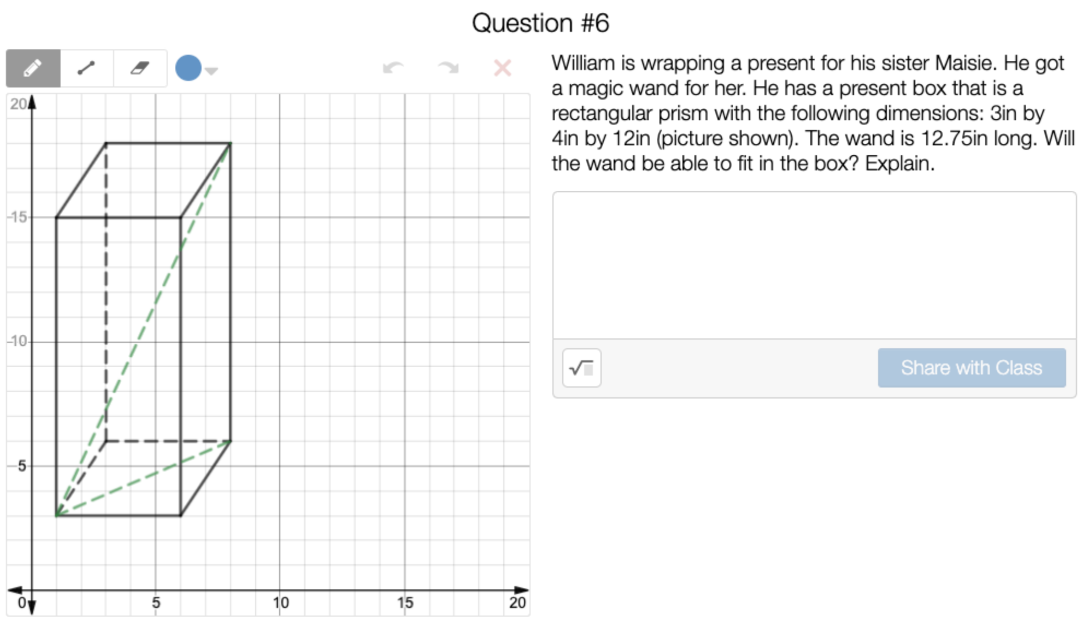
Figure 5: An example of a problem on Lesson 4



Lesson 5

These final lesson goals were to make PSTs think about possible application of the Pythagorean theorem problem of coordinate planes, such as those for the calculation of distance between two points. Also, to determine if specific provided lengths formed a right triangle (the inverse of the Pythagorean theorem), and finally being able to apply the theorem to solve real-world problems. The latter is exemplified in figure 6 below.

Figure 6: A real-world problem that used the Pythagorean theorem on its solution

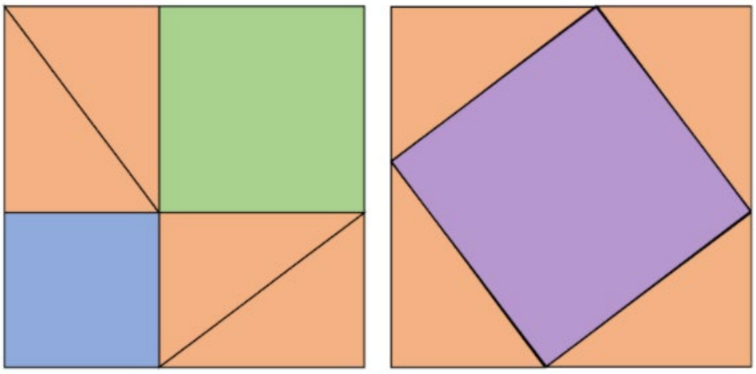


APPENDIX B

Question 1. Can you state the relationship between the legs of a right triangle?

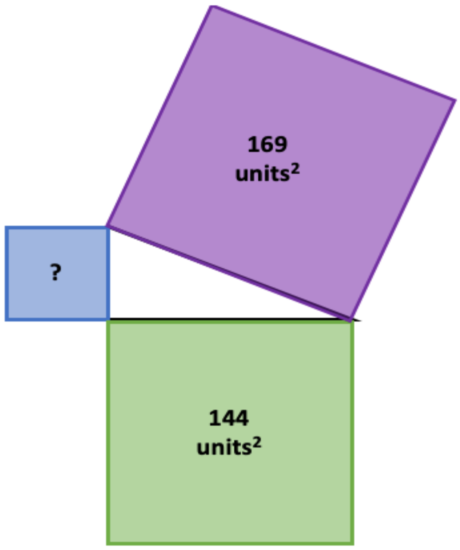
Cognitive Demand	Low-level demand, because it involves exact reproduction of previously learned procedure
Full Credit (2 points)	• $a^2+b^2=c^2$ or related answers that expresses the formula meaning
Partial Credit (1 point)	• a+b=c • The hypotenuse is larger than the legs
No Credit (0 points)	• Other responses. • Missing .

Question 2. Can you use the picture below as a guide to prove the Pythagorean Theorem?



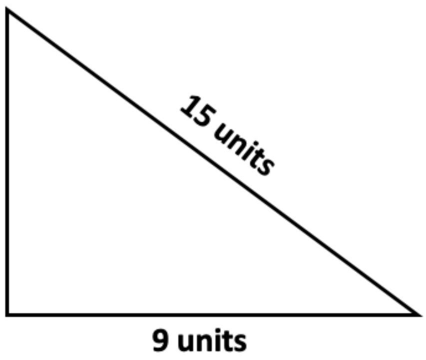
Cognitive Demand	High-level demand, because it requires students to explore and understand the nature of mathematical concepts, processes, and relationships
Full Credit (4 points)	• Students can state that the area of the purple square is equal to the area of the blue square plus the green square by using algebraic and geometrical relationships.
Partial Credit (2 points)	• Student can see the relationship between the squares, but failed to provide a full explanation of his/her answer.
No Credit (0 points)	• Other responses. • Missing .

Question 3. In the picture below, what is the area of the blue square? How do you know?



Cognitive Demand	High-level demand, because it requires students to engage with the conceptual ideas that underlie the procedures to complete the task
Full Credit (4 points)	• 25 because it is the area of the purple minus the area of the green • Or 5^2
Partial Credit (2 points)	• 5, mistaken forgetting to square the length of that leg.
No Credit (0 points)	• Other responses. • Missing .

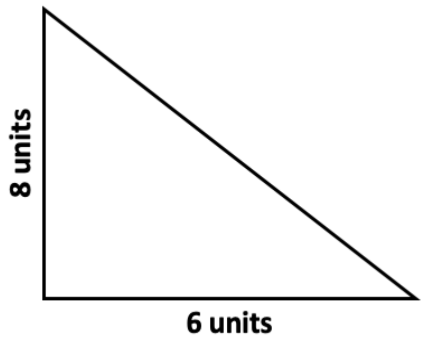
Question 4. Determine the length of the unknown side in the diagram below. If the solution isn't a perfect square, you can use the words, "the square root of" in replace of the symbol typically used.



Cognitive Demand	Low-level demand, because it specifically calls for the use of a procedure
Full Credit (2 points)	• $A^2=15^2+9^2$ or similar answers • Root square of 144 • 12
Partial Credit (1 point)	• 6, student mistakenly use Pythagorean theorem as a+b=c • 144

	• Root square of 12
No Credit (0 points)	• Other responses. • Missing .

Question 5. Determine the length of the unknown side in the diagram below. If the solution isn't a perfect square, you can use the words, "the square root of" in replace of the symbol typically used.

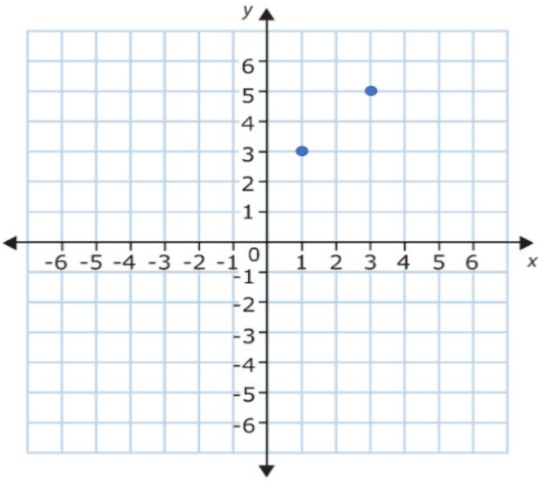


Cognitive Demand	Low-level demand, because it specifically calls for the use of a procedure
Full Credit (2 points)	• $C^2=8^2+6^2$ or similar answers • Root square of 100 • 10
Partial Credit (1 point)	• 48, student mistakenly use Pythagorean theorem as a+b=c • 100 • Root square of 10
No Credit (0 points)	• Other responses. • Missing .

Question 6. Can the following lengths be used for the legs of a right triangle? How do you know? 7 cm, 8 cm, and 9cm

Cognitive Demand	High-level demand, because it requires students to engage with the conceptual ideas that underlie the procedures to complete the task
Full Credit (4 points)	• No, because 81 not equal to 113
Partial Credit (2 points)	• Students says no but doesn't explain his/her thoughts
No Credit (0 points)	• Other responses. • Missing .

Question 7. What is the distance between the two points in the figure below? How did you determine your solution? If the solution isn't a perfect square, you can use the words, "the square root of" in replace of the symbol typically used.



Cognitive Demand	High-level demand, because it requires students to engage with the conceptual ideas that underlie the procedures to complete the task
Full Credit (4 points)	• $C^2=8$, root square of 8 or similar answers
Partial Credit (2 points)	• 4 mistakenly considering the theorem to be a+b=c. or 8 forgetting to take the root square
No Credit (0 points)	• Other responses. • Missing .

Question 8. Will is building a treehouse. He puts the posts in the four corners. Will wants to determine if the base of the tree house is "square" (the corner forms a 90 degree angle). He measures the width of the tree house and gets 12 feet. Then he measures the depth and gets 5 feet. What would he need to measure next? What would that measurement need to be for the base of the tree house to be "square" (the corner forms a 90 degree angle)?

Cognitive Demand	High-level demand, because it requires from students considerable cognitive effort
Full Credit (4 points)	• The diagonal, and the answer would need to be root square of 169 or 13
Partial Credit (2 points)	• Respond only one question • Says the measure would be 169 or root square of 13
No Credit (0 points)	• Other responses. • Missing.